THE 2016 GRADUATE STUDENT MOCK AMS CONFERENCE

Schedule for Wednesday July 27

(All talks will be held in Dawson 110)

- 9:15-9:30 coffee, tea, refreshments (in the Matrix)

- 9:30-9:50 Luca Schaffler
- 9:55- 10:15 Xian Wu
- 10:20 - 10: 40 Clay Mersmann
- 10:45-11:05 Daniel McKenzie
- 11:10-11:30 Eric Perkerson

- 11:30-1:00 Lunch

- 1:00-1:20 Abe Varghese
- 1:25-1:45 Zerotti Woods
- 1:50-2:10 Zachary Emrich
- 2:15-2:35 Riley Ellis
- 2:40-3:00 Ernest Guico

- 3:00-3:20 coffee, tea, refreshments (in the Matrix)

- 3:20-3:40 Saurabh Gosavi
- 3:45-4:05 Noah Lebowitz-Lockard
- 4:10-4:30 Kubra Benli
- 4:35 - 4:55 Jordan Clark
- 5:00-5:20 Marko Milosevic

Date: July 26, 2016.

Schedule for Thursday July 28

(All talks will be held in Dawson 110)

- 9:45-10:00 coffee, tea, refreshments (in the Matrix)

- 10:00 - 10:20 M. Barowsky, W. Damron, A. Mejia, F. Saia, N. Schock
- 10:25 - 10:45 K. Hess, E. Stamm, T. Warren
- 10:50 - 11:10 Hans Parshall
- 11:15-11:35 Peter Woolfitt

- 11:35-1:00 Lunch

- 1:00-1:20 Jason Joseph
- 1:25-1:45 Bret Stevenson
- 1:50-2:10 James Taylor
- 2:15-2:35 Nate Zbacnik
- 2:40-3:00 Jun Zhang

- 3:00-3:20 coffee, tea, refreshments (in the Matrix)

- 3:20-3:40 Ziqing Xiang
- 3:45-4:05 Bill Olsen
- 4:10-4:30 Brian Bonsignore
- 4:35-4:55 Phong Luu
- 5:00 - 5:20 George Slavov

1. Titles and Abstracts

1.1. **Madeleine Barowsky (Wellesley), William Damron (Davidson), Andres Mejia (Bard), Frederick Saia (Tufts), Nolan Schock (Cal Poly San Luis Obispo).** Classically Integral Quadratic Forms Excepting at Most Two Values

Abstract. The study of universal quadratic forms has long been of interest to number theorists. Quadratic forms that are almost universal, failing to represent some finite set of numbers, are less explored, though Halmos (1938) published a list of diagonal quaternary forms that except just one value. Using analytic and computational methods from Bhargava and Hanke, we provide an enumeration of and proofs for the 65 possible value pairs and the 9 possible singleton values that a classically integral, positive-definite, quaternary quadratic form may except. Additionally, we provide the number of forms up to equivalence in each family of almost universality.

1.2. **Kubra Benli.** Swiching digits in prime numbers.

Abstract. The infinitude of primes that change into a composite number after any change in the digits is proved by Erdos. Next question to ask is can we ensure all resulting composite numbers after changing the digits of some primes having "an amount" of distinct prime factors. We will present a current result on this question.

1.3. **Brian Bonsignore.** n-Equivalence of DGAs in Rational Homotopy Theory

Abstract. n rational homotopy theory, we consider two spaces to be equivalent if there is a zig-zag of maps inducing isomorphisms on their rational cohomology algebras. By an equivalence of categories, we can also study rational homotopy theory using differential graded algebras (dga) with the same notion of equivalence. In this talk, I'll discuss some of the consequences of weakening this full equivalence to a *n-equivalence*, where we only require dga maps inducing isomorphisms on cohomology through degree n and an injection in degree $n+1$. In particular, although $n+1$ degree cohomology is not an invariant of n-equivalence, a subset of "homotopically decomposable" elements of $n+1$ cohomology is invariant.

1.4. **Jordan Clark.** On Generalizations of the Second Euclid-Mullin Sequence

ABSTRACT. he second Euclid-Mullin sequence is an infinite sequence of primes related to Euclids proof of the infinitude of primes. Booker showed this sequence misses infinitely many primes. Pollack and Treviño showed the same thing with a completely elementary proof. We adapt the Pollack and Treviño argument to show certain related sequences also miss infinitely many primes.

1.5. **Riley Ellis.** Clifford Algebras and Their Representations

ABSTRACT. Instead of just making the number -1 into a square, what the Complex Numbers really do is make the quadratic form $Q(x) = -x^2$ into one. Similarly, the famous Quaternions turn out to be the "smallest" ($\mathbb{R}$-)algebra in which $Q(x, y) = -x^2 - y^2$ is a square. Clifford Algebras generalize this idea, and have deep connections to many areas both inside and outside of pure mathematics. In this talk I will focus on matrix representations of Clifford Algebras and their connection—in fact, equivalence—to the existence of a certain type of module ("linear maximal Cohen-Macaulay modules"), the existence of which is a question of high interest and activity in Commutative Algebra.

1.6. **Zachary Emrich.** The Mathematics of Signal Reconstruction

ABSTRACT. The idea behind compressive sensing is that a sensing device can capture all the information of a sparse signal and store it as a much smaller amount of data. Then computationally one is able to reconstruct the original signal using the stored information. This leads to two basic questions. In what situations is recovery possible? And given that recovery is possible, what is an ecient method of recovery?

In our situation, the sensing device is a matrix and our goal is to recover the most sparse solution to an underdetermined linear system, $Ax = b$. We nd that numerically the discrete fourier transform (DFT) matrix satises sucient conditions to support accurate recovery of the original sparse data. Thus using the DFT matrix as our sensing matrix, A, allows for computationally fast recovery of the original sparse data.

Numerical experiments are carried out to show the eciency and accuracy of the sparse recovery using the DFT matrix.

1.7. **Saurabh Gosavi.** (Title TBA)

Abstract. A factorization domain is an integral domain in which every non-zero, non-unit can be (not necessarily uniquely) factorized into product of irreducibles. Factorization domains having finitely many irreducibles up to associates are called Cohen Kaplansky domains, in honor of Cohen and Kaplansky who initiated their study in 1945. Later, D. D Anderson and J. L Mott largely contributed to their study and among other things provided fourteen different characterizations of these domains! In this talk, we will use some of their results and a bit of additive number theory to show that for every $n \geq 3$, there exist Cohen Kaplansky domains having exactly n number of (non-prime) irreducibles up to associates improving a conditional result of Coykendall and Spicer.

1.8. **Ernest Guico.** Twisted Forms and Cohomology

Abstract. In this talk we will consider vector spaces equipped, perhaps, with some additional structure. In particular we will discuss their so called "twisted forms" and explore how these relate to a certain cohomology set. Hilbert's theorem 90, as well as the Brauer group, arise as examples of interest.

1.9. **Kylie Hess (Rose-Hulman), Emily Stamm (Vassar), Terrin Warren (UGA).** When is $a^n + 1$ a sum of two squares?

Abstract. Using Fermat's two squares theorem and properties of cyclotomic polynomials, we prove assertions about when numbers of the form $a^n + 1$ can be expressed as the sum of two integer squares. We prove that $a^n + 1$ is the sum of two squares for all $n \in \mathbb{N}$ if and only if a is a perfect square. Also, if a is even and n is an odd integer so that $a^n + 1$ is the sum of two squares, then $a^p + 1$ is the sum of two squares for all primes $p|n$.

1.10. **Jason Joseph.** The Alexander Invariant

Abstract. The Alexander polynomial is one of the oldest and most famous invariants of knot theory. Alexander's original definition in 1928 involved a module structure on the homology of a special covering space of the knot complement. We will construct this covering space and define the Alexander invariant for generalized knots, then conclude with an example of a knot which has no Alexander polynomial.

1.11. **Noah Lebowitz-Lockard.** Subatomic Domains

Abstract. Algebraists traditionally consider "atomic" domains (ones in which every element is the product of irreducible elements). Just last year, Jason Boynton and Jim Coykendall created two new properties which are weaker than atomicity. They called these properties "almost atomicity" and "quasi-atomicity". In this talk, we discuss these properties, along with some new properties that relate to Furstenberg's "topological" proof of the infinitude of primes.

1.12. **Phong Luu.** An Optimal Stopping Problem

Abstract. Let X_t be an Itô diffusion on $\mathbb{R}^n$ and let g be a reward function. We want to find an optimal stopping time $\tau^* = \tau^*(x, \omega)$ for X_t such that

$$E^x[g(X_{\tau^*})] = \sup_{\tau} E^x[g(X_\tau)] \tag{1}$$

for all $x \in \mathbb{R}^n$. We may regard X_t as the state of a game at time t, and each ω corresponds to one sample of the game. For each time t, we have the option of stopping the game, thereby obtaining the reward $g(X_t)$, or continue the game in hope that stopping it at a later time will give a bigger reward. So, among all possible stopping times τ, we are asking for the optimal one, τ^*, which gives the best result in the long run, i.e., the biggest expected reward in the sense of (1).

1.13. **Andrew Maurer.** Cohomology and Support Varieties

Abstract. Perhaps the most useful object in the representation theory of finite groups is the cohomology ring. By forgetting nilpotent information, the cohomology ring becomes a finitely generated commutative algebra, which opens up an entire world of algebraic geometry. The spectrum of the cohomology ring is an affine algebraic variety, whose subvarieties and geometric properties capture the rich representation theory of our group. In this talk I will give a soft introduction to these concepts and some of the more exciting results in the field.

1.14. **Daniel McKenzie.** An Application of Compressed Sensing to the Graph Clustering Problem

Abstract. Given a graph G with vertex set V and edge set E, it is frequently important to partition V into subsets $C_1, \ldots, C_k$ such that there are 'many' edges between vertices in the same subset and 'few' edges between vertices in different subsets.

Spectral graph theory provides an elegant solution to this problem, namely one finds the first k eigenvectors, $\mathbf{v}_1, \ldots, \mathbf{v}_k$ of a certain matrix $\mathbf{L}$ which is naturally associated to G. Loosely, $\mathbf{v}_i$ should be 'close' to the indicator vector of C_i, $\mathbf{1}_i$, which has a one as its j-the entry if and only if the j-th vertex belongs to C_i, and a zero otherwise. Unfortunately computing eigenvectors takes $\mathcal{O}(n^3)$ operations, where $n = |V|$, and thus this approach cannot be applied to truly large graphs.

However, if one assumes that the vertices in G have bounded degree (a reasonable assumption in most cases of interest), one can show that the indicator vectors are sparse. So, one can adapt $\mathcal{O}(n^2)$ algorithms from Compressed Sensing to solve for them directly. In this talk we shall discuss this (novel) idea and present analytic and experimental evidence that it is at least as effective as existing clustering algorithms.

1.15. **Clay Mersmann.** An Adaptive Mesh Method for Numerical Solutions of PDEs

Abstract. We study a globally adaptive mesh for numerically solving PDEs through the use of bivariate splines. Based on a previous solution, we estimate the location of sharp changes and at spots; then, we add vertices to and subtract vertices from the existing mesh to generate a new one. Since the new triangulation is generated by the Delaunay method, it oers several advantages over locally adjusted adaptive mesh methods.

Together with the exibility of the spline method for arbitrary degree and smoothness of solutions, our approach provides a more eective way to solve PDEs numerically.

1.16. **Marko Milosevic.** Homomorphic Encryption

Abstract. Homomorphic Encryption is a form of encryption that allows operations to be preformed upon the cipher-text while preserving privacy. Homomorphic Encryption that allows for arbitrary computation was a long-standing open problem in cryptography until it was solved by Craig Gentry in 2009. I will present a short survey of the problem.

1.17. **Bill Olsen.** Knots: What are they and why do we care?

Abstract. The study of knots has produced an enormous volume of mathematical theory. In this presentation we will survey this eld by focusing rst on classical knot invariants such as the unknotting number of a knot, the Seifert genus of a knot, the slice genus of a knot, and then transitioning to a more recent invariant: the Heegaard Floer homology of a knot. We draw connections between these invariants, and well unveil the knot theory hiding underneath 3 and 4-manifold topology.

1.18. **Hans Parshall.** Spherical configurations in dense sets

Abstract. A finite set X in $\mathbf{R}^k$ is called Ramsey if for any r colors, there is some large enough dimension $d = d(X, r)$ for which any r-coloring of $\mathbf{R}^d$ results in a monochromatic congruent copy of X. In 1973, Erdős, Graham, Montgomery, Rothschild, Spencer, and Straus proved that all Ramsey sets are spherical, and they conjectured that all finite spherical sets were indeed Ramsey. We will discuss the state of this conjecture and its analogue in $\mathbf{F}_q^d$.

1.19. **Eric Perkerson.** Sparse Subspace Clustering with Applications

Abstract. The sparse subspace clustering algorithm of Elhamifar and Vidal groups data points from high-dimensional spaces that belong to the same subspace (or affine space). The algorithm relies on the fact that, of the representations of each data point in terms of linear or affine combinations of other points, a sparse representation corresponds to a representation in terms of points from the same subspace. I describe how the algorithm works and look into applications of this algorithm to computer vision, including facial recognition and motion segmentation.

1.20. **Luca Schaffler.** Polyhedral subdivisions of the unit cube

Abstract. In this talk we study the polyhedral subdivisions of the 3-dimensional unit cube by systematically enumerating them and proving that they are all regular. At the same time, we explain how these facts in convex geometry are related to moduli theory in algebraic geometry. To conclude, we discuss similar questions for the 4-dimensional unit cube.

1.21. **George Slavov.** Bivariate Spline Solution to a Class of Reaction-Diffusion Equations

Abstract. I will present a method of solving a time-dependent partial differential equation, which arises from classic models in ecology concerned with a species population density over two dimensional domains. The species experiences population growth and diffuses over time due to overcrowding. Population growth is modeled using logistic growth with Allee effect. This work introduces the concept of discrete weak solution and establish theory for the existence, uniqueness and stability of the solution. Bivariate splines of arbitrary degree and smoothness across elements are used to approximate the discrete weak solution. More recent efforts focus on modeling the interaction of multiple species, which either compete for a common resource or one predates on the other.

1.22. **Bret Stevenson.** Barcodes, Morse theory, and Hamiltonian Floer theory

Abstract. Broadly speaking, a *barcode* is a collection of intervals which together depict the changes in homology as some parameter varies. Though this machinery was originally developed in the context of topological data analysis, it has since been adapted to Morse and Hamiltonian Floer theory. This talk will start by introducing barcodes and their continuous nature (via illuminating examples from Morse theory) and conclude by briefly discussing how barcodes can be used to answer questions from Hamiltonian dynamics. Plenty of pictures will feature.

1.23. **James Taylor.** Compactifying Topological Spaces

Abstract. We will be discussing different methods of compactifying topological spaces: the one-point compactification, n-point compactifications (finite n), and the Stone-Čech compactification. We will examine when they exist, why they are useful, and touch on some interesting examples.

1.24. **Abe Varghese.**

$$\begin{bmatrix} \text{Montgomery} & \text{Polya} & \text{Von Neumann} & \text{Dyson} & \text{Hilbert} \\ \text{Marchenko} & \text{Wishart} & \text{Wigner} & \text{Pastur} & \text{Goldstine} \end{bmatrix}$$

Abstract. What has these random matrix of personalities to do with each other? Come find it out in the talk!

1.25. **Zerotti Woods.** A brief Look at the SIR Model

Abstract. In this talk I will introduce a very simple epidemiological model called the SIR Model. I will discuss a few examples that will explain when this model is useful, some of its limitations and discuss some stability analysis.

1.26. **Peter Woolfitt.** Frighteningly Large Numbers

Abstract. The busy beaver sequence BB(n) grows at an absurd rate (in a precise sense) and is of fundamental importance to theoretical computer science. In this talk, I will give some sense of how large these numbers are and discuss their relation to the halting problem. This will imply that for some nite number n, we cannot prove the value of BB(n) under Zermelo-Fraenkel set theory with the axiom of choice. I will also discuss the recent work of Adam Yededia and Scott Aaronson giving bounds on this number.

1.27. **Xian Wu.** An Application of GIT Quotient in Birational Geometry.

Abstract. I will talk about a 5 page result by Hu and Keel. They gave a short proof of Wlodarczyks weighted factorization theorem using geometric invariant theory quotients.

1.28. **Ziqing Xiang.** Support varieties for Hecke algebras of type A

Abstract. Support varieties for Hecke algebras detect natural homological properties like the complexity of modules. They have a natural description in an affine space where computations are tractable. Calculations for permutation modules and Young modules are presented.

1.29. **Nathaniel Zbacnik.** Ramified Covers of Riemann Surfaces

Abstract. This will be an introductory look at building Riemann Surfaces that leads us to ramified covers and the Riemann-Hurwitz formula. We will discuss the sorts of enumerative questions that can be asked when we attempt to count maps of a certain degree from one surface to another.

1.30. **Jun Zhang.** Bott periodicity theorem.

Abstract. Bott periodicity theorem describes a periodicity in the homotopy groups of classical groups. It is fundamentally important for algebraic topology and also the study of (topological) K-theory. In this talk, I will introduce sufficient background to understand this phenomenon. Moreover, I will use language of K-theory to rewrite this theorem and state a more general theorem which plays an important role of the development of K-theory.